

## THE REST OF THE CALCULATION

by Robert Beyma

The *THIRD REICH* combat results table bears some interesting analysis. The 2-1 is probably the most important attack in the game. While not a guaranteed victory, it is so close to one that most players take 2-1's for granted along their paths of conquest. The numerous counterattacks make analysis a bit more complicated than on conventional CRT's. We will be primarily concerned with the probability of victory and the expected attacker losses. The defender will always be eliminated except on an A-Elim roll.

First, let us examine the probability of each possible result. The initial iteration (the first attacker roll followed by the first defender roll) produces the following results:

P(full exchange)	= 1/6	= 1/6
P(CA exchange)	= (1/3*1/3) + (1/6*1/3)	= 1/6
P(A Elim)	= (1/6*1/6)	= 1/36
P(D Elim)	= 1/3 + (1/3*1/2)+(1/6*1/6)	= 19/36
P(no result)	= (1/3 * 1/6)+1/6*1/3	= 1/9

When you get a no result, you must go through a second iteration and so on. Mathematically this becomes

$$P(\text{total}) = P(\text{initial}) [1 + 1/9 + 1/81 + \dots + 1/9^m]$$

This expression reduces to  $P(\text{total}) = 1.125 P(\text{initial})$ . Therefore, the actual probabilities become:

P(full exchange)	= 1/6	*
1.125	=0.1875	
P(CA exchange)	= 1/6 * 1.125	=0.1875
P(A Elim)	= 1/36 * 1.125	=0.03125
P(D Elim)	= 19/36 * 1.125	=0.59375
		1.00000

Thus, the actual probability of losing a 2-1 becomes 1:32 and not 1/36 as one might initially expect. Hence we see why the 2-1 is so often taken for granted.

The next step is to determine the expected losses. Actually, this is quite easy to do once the probabilities have been calculated. Merely figure up how many BRP's you would lose on each type of result.

A= losses on a full exchange  
 B= losses on a CA exchange  
 C= losses on an A Elim

$$\text{Expected Losses} = 0.1875 (a+b) + .03125 (c)$$

An interesting feature is that the full exchange result will usually contribute more to the expected losses than the A Elim.

The 3-1 is a guaranteed attack with about 35% less expected losses than the 2-1. The probabilities of each result and the expected losses can be computed in a similar manner to the 2-1. These results are:

P(full exchange)	= 0.1714
P(CA exchange)	= 0.1143
P(D Elim)	=0.7143
	1.0000

$$\text{Expected Losses} = 0.1714 (a) + 0.1143 (b)$$

While the 3-1 is clearly superior to the 2-1, the offensive player in *THIRD REICH* can rarely afford the luxury of 3-1's on his major attacks. Limited resources, too few hexes from which to attack, and enemy defensive air will usually limit the major attacks to 2-1 or even 1-1.

The 1-1 attack is a bit more difficult to analyze because the CA odds may vary depending on the factors involved. Usually the CA will be at 1-2 or less since the defender is nearly always at least doubled on defense. This almost invariably produces a Victory for the attacker. The rough spot on the 1-1 is that there is at least a 1/6 chance of an A-Elim. Full exchanges, a 1/3 chance, tend to be very costly. Be careful to include an extra piece in your 1-1's so that you will have a piece left over to advance into the hex. This leaves about a 5/6 chance of winning the

terrain. Not a bad chance but expect a lot more losses than on 2-1's and 3-1's.

Let's use the Polish defense problem presented in contest #69 (Vol. 12, No. 4) as an example for our computations. Against this defense an initial 2-1 attack may be made against Warsaw. The attacking force will consist of two 3-3 infantry units plus 18 air factors. The two Polish air units are countered. For optimum results a 5-1 against an adjacent 1-3 is set up. This attack will only have to be made if both 3-3's are lost in a full exchange. An armor unit could then exploit into an unoccupied Warsaw.

$$\begin{aligned} P(\text{victory}) &= 1 - P(A \text{ Elim}) = 1 - 0.03125 = 0.96875 \\ \text{Expected Losses} &= .1875(24+6) + 0.03125(60) + \\ &0.1875(1/6 \cdot 3) = 7.59 \end{aligned}$$

These results are slightly less favorable than the solution featured in the magazine because of the increased accuracy of the calculations due to the iterations.

Let's also examine a slightly different defense. Move the 1-3 from Brest-Litovsk to the hex northeast of Warsaw. The optimum method of attacking this defense is to 2-1 the 2-3 and to attack Warsaw with a 3-1 exploitation attack. Note that the 3-1 attack will not be made if the 2-1 fails. The best tactical disposition is a 4-6 + 4 air factors against the 2-3 and (3) 4-6's and 12 air factors against Warsaw at 3-1.

$$P(\text{victory}) = 0.96875 \text{ (or } 31/32)$$

$$\begin{aligned} \text{Expected Losses} &= 0.1875(12+6) + 0.03125(20) + \\ &0.96875 [0.1714(16) + 0.1143(8)] = 7.54 \end{aligned}$$

The results are consistent with the contest solution. However, there is more to these calculations than one might have thought. While the back door approach is 0.05 BRP cheaper for the Germans it does have two advantages. First, it forces commitment of the German armor to the east if Poland is to be conquered on turn 1. Second, the probability of no German losses in taking Poland is reduced. The moral is that there are frequently other considerations besides expected BRP losses.

I hope that I have given some insight into analyzing the *THIRD REICH* combat results table. While one does not normally stop a game to make precise calculations, players can develop a feel for the Fisk and expected losses inherent in a certain line of play. The 2-1 is the attack to use to get the job done. 3-1's are nice when you can afford them. 1-1's are not bad but save them for really important situations when you cannot get a 2-1.